LAWS OF PROPAGATION OF AN INSTANTANEOUS HEAT PULSE IN A FLUIDIZED BED

A. I. Tamarin

Inzhenerno-Fizicheskii Zhurnal, Vol. 10, No. 1, pp. 72–77, 1966

UDC 536.248 + 536.222

A function is obtained representing the effect of an instantaneous heat pulse in a fluidized bed in the presence of heat losses by way of the gas flow and through the external walls of the cylindrical apparatus.

In a bed of finely dispersed material fluidized by a gas heat transfer takes place mainly via the solid particles, the volume heat capacity of which exceeds that of the gas by several orders. With the heat loss from the system by way of the gas flow taken into account, the law of heat transfer can be described by the following differential equation:

$$\frac{\partial \Theta}{\partial \tau} = a_{ef}\nabla^2 \Theta + bU \operatorname{grad} \Theta, \tag{1}$$

where

$$b = \gamma c \varepsilon / \gamma_{pa} c_{pa}(1 - \varepsilon).$$

Let us consider a cylindrical apparatus containing a fluidized bed. At the upper boundary of the bed we produce a heat pulse by rapidly pouring into the bed a small quantity of hot particles of the same material. This generates in the system of thermal wave directed downwards, opposite to the direction of gas flow. The gas passing through the bed continuously carries away some of the heat. At the same time, part of the thermal energy introduced by the hot particles is removed from the bed through the external cylindrical surface. Both phenomena combine to facilitate a more rapid damping of the heat pulse in the system compared with a system in which heat losses are practically absent. The problem is to find a function representing the effect of the instantaneous heat pulse on the system considered; using this fucntion, it should be possible to obtain experimentally the effective thermal diffusivity of the fluidized bed from the distance to the heat source and the time needed to reach the temperature maximum [1].

Let us now set up the boundary and initial conditions.

The initial temperature of the bed is constant and equal to the temperature of the surrounding air:

$$t - t_0 = \Theta = 0 \qquad \text{at} \quad \tau = 0. \tag{2}$$

The temperature at the lower boundary of the bed remains constant at its inital value;

$$t - t_0 = \Theta = 0 \qquad \text{at} \quad z = \infty. \tag{3}$$

Heat transfer takes place through the external cylindrical surface to the surrounding air:

$$\frac{d\Theta}{dr} = -\frac{\alpha}{\lambda_{ef}}\Theta \qquad \text{at} \quad r = R. \tag{4}$$

In the case being considered (1) it is convenient to use cylindrical coordinates. If the z axis coincides with the axis of the apparatus and the gas flows only in the direction of this axis ($U_{pa} = 0$), we have

$$\frac{\partial \Theta}{\partial \tau} = a_{ef}\left(\frac{\partial^2 \Theta}{\partial r^2} + \frac{1}{r}\frac{\partial \Theta}{\partial r} + \frac{\partial^2 \Theta}{\partial z^2}\right) + bU\frac{\partial \Theta}{\partial z}. \tag{5}$$

It can be assumed that the solution of this equation is a product of two functions, the former being dependent on the variables r, τ, z and the latter varying with the coordinate z only. The latter function characterizes the rate of damping of the heat pulse due to the heat carried away by the counterflow of gas. It sharply decreases with distance from the point of action of the instantaneous heat source. We seek the solution of (5) in the following form:

$$\Theta = C\exp[-bU(z - z_0)/2a_{ef}]\, W(\tau, r, z). \tag{6}$$

Substitution of (6) into (5) gives a differential equation which can be used to find the function W:

$$\frac{1}{a_{ef}}\frac{\partial W}{\partial \tau} = \frac{\partial^2 W}{\partial r^2} + \frac{1}{r}\frac{\partial W}{\partial r} + \frac{\partial^2 W}{\partial z^2} - \frac{(bU)^2}{2a_{ef}^2} W. \tag{7}$$

We now employ the Fourier method. We represent the solution of Eq. (7) as a product of two functions:

$$W = R(r)V(\tau, z). \tag{7a}$$

By solving the Sturm-Liouville problem, we obtain a system of two equations:

$$\frac{\partial V}{\partial \tau} - a_{ef}\frac{\partial^2 V}{\partial z^2} + \frac{(bU)^2}{2a_{ef}}V + \lambda^2 V a_{ef} = 0, \tag{8}$$

$$\frac{\partial^2 R}{\partial r^2} + \frac{1}{r}\frac{\partial R}{\partial r} + \lambda^2 R = 0. \tag{9}$$

Equation (9) is a form of Bessel's equation. Its particular solution has the following form [2]:

$$R(r) = CI_0(\lambda, r). \tag{10}$$

The boundary condition (3) gives a relation which determines the eigenvalues of the parameter λ:

$$\lambda R\,\mathrm{Bi} = I_0(\lambda, R)/I_1(\lambda, R). \qquad (11)$$

According to the conditions of the problem, at the instant τ_0 an instantaneous plane heat pulse acts at the upper boundary z_0 of the bed. We now consider Eq. (8). In this case its solution is given by the Green's function V = G. Equation (8) can be symbolically written in the form LV = 0, in which case the condition for the operation of the instantaneous heat source with coordinates z_0, τ_0 becomes

$$L\,G = -\delta(z - z_0, \quad \tau - \tau_0).$$

Using the separability property of the delta function and also introducing a symbol for the inverse operation L^{-1}, we obtain

$$G = -L^{-1}\delta(z - z_0)\,\delta(\tau - \tau_0).$$

Let us now represent the delta function as a Fourier integral [3]:

$$\delta(z - z_0)\,\delta(\tau - \tau_0) = \frac{1}{(2\pi)^2}\int\int_{-\infty}^{+\infty} \exp[ik_1(z - z_0) + ik_2(\tau - \tau_0)]\,dk_1\,dk_2$$

and using the operator L^{-1}, we obtain

$$G = \frac{-1}{(2\pi)^2}\int\int_{-\infty}^{+\infty}\left[\lambda_i^2 a_{ef} + a_{ef}k_1^2 + \frac{(bU)^2}{2a_{ef}} + ik_2\right]^{-1} \exp[ik_1(z - z_0) + ik_2(\tau - \tau_0)]\,dk_1\,dk_2. \qquad (12)$$

We now integrate Eq. (12), first with respect to the variable k_2, for which purpose we use the theory of residues and the fact that the integrand function has one singular point:

$$ik_2 = -a_{ef}k_1^2 - (bU)^2/2a_{ef} - \lambda_i^2 a_{ef}.$$

The integral of (12) with respect to the variable k_2 is equal to the residue at this point.

After some manipulation we obtain

$$G = -\frac{1}{\pi}\exp\left[(\tau - \tau_0)(-\lambda_i^2 a_{ef} - (bU)^2/2a_{ef})^2\right]\int_{-\infty}^{+\infty}\cos k_1(z - z_0)\exp[-k_1^2 a_{ef}(\tau - \tau_0)]\,dk_1.$$

Remembering that the integral on the right-hand side is [4]

$$\sqrt{\pi/a_{ef}(\tau - \tau_0)}\exp[-(z - z_0)^2/4a_{ef}(\tau - \tau_0)],$$

we finally obtain

$$G = -\frac{1}{\sqrt{a_{ef}\pi(\tau - \tau_0)}}\exp\left[(\tau - \tau_0)\left(-\lambda_i^2 a_{ef} - \frac{(bU)^2}{2a_{ef}}\right) - \frac{(z - z_0)^2}{4a_{ef}(\tau - \tau_0)}\right]. \qquad (13)$$

We now employ the principle of superposition of solutions. From (5), (7a), (13) we determine the variation of the temperature at the point z when at time τ_0 an instantaneous heat pulse acts at the point z_0. In order to simplify the calculations, we set $\tau_0 = 0$, $z_0 = 0$. Then

$$\Theta = \frac{C}{\sqrt{a_{ef}\pi\tau}}\exp\left(-\frac{bUz}{2a_{ef}}\right)\sum_{i=1}^{\infty} I_0(\lambda_i, r) \times \exp\left[\tau\left(-\lambda_i^2 a_{ef} - \frac{(bU)^2}{2a_{ef}}\right) - \frac{z^2}{4a_{ef}\tau}\right]. \qquad (14)$$

By differentiating the function (14) with respect to time and equating the derivatives to zero, we determine its maximum. After simplification we have

$$\sum_{i=1}^{\infty} I_0(\lambda_i, r)[\exp(-2\lambda_i^2 a_{ef}\tau_m)] \times \left[\left(2\lambda_i^2 a_{ef} + \frac{(bU)^2}{a_{ef}}\right)\tau_m^2 + \tau_m - \frac{z^2}{4a_{ef}}\right] = 0. \qquad (15)$$

The eigenvalues of the parameter λ_i (the roots of Eq. (11)) continuously increase with increasing i; this results in a rapid reduction of the factor $I_0(\lambda_i, r)\exp(-2\lambda_i^2 a_{ef}\tau_m)$ which tends to zero as $\lambda_i \to \infty$. The series (15) converges rapidly; therefore for finding the time of the temperature maximum it is quite sufficient to use the first term of the series only. Substitution of the value of the first root of Eq. (11) $R^2\lambda_1^2 = 2\mathrm{Bi}$ into (5) gives, after manipulation,

$$\tau_m^2(4a_{ef}\mathrm{Bi}/R^2 + (bU)^2/a_{ef}) + \tau_m - z^2/4a_{ef} = 0.$$

Since the negative solution contradicts the physical meaning of the problem, we write the expression for the positive root:

$$\tau_m = \frac{1}{2}\left[4\frac{a_{ef}\mathrm{Bi}}{R^2} + \frac{(bU)^2}{a_{ef}}\right]^{-1} \times \left[\sqrt{1 + \left[4\frac{a_{ef}\mathrm{Bi}}{R^2} + \frac{(bU)^2}{a_{ef}}\right]\frac{z^2}{a_{ef}}} - 1\right]. \qquad (16)$$

The relation thus obtained makes possible the determination of the time when the maximum temperature occurs at a given point with coordinate z after operation of an instantaneous heat pulse at the origin of the coordinate system, provided that the heat losses take place via the fluidizing gas and through the external cylindrical surface. Without heat losses from the system (Bi → 0 and U → 0), the removal of the indeterminacy yields the well-known relation, which is used for the experimental determination of the thermal conductivity:

$$\tau_m^* = z^2/4a_{ef}. \qquad (17)$$

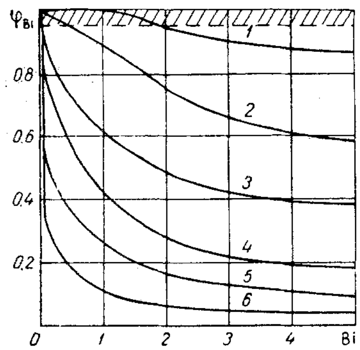

Fig. 1. Dependence of the time needed to reach the temperature maximum on Bi and the height z/R: 1–6) z/r = 0.2; 0.5; 1; 2; 4; 10.

Comparing (16) and (17), we calculate the correction for the temperature maximum due to loss of heat from the system:

$$\varphi = \tau_m/\tau_m^* = 2\{[4a_{ef}Bi/R^2 + (bU)^2/a_{ef}]z^2/a_{ef}\}^{-1} \times$$
$$\times [\sqrt{1 + [4a_{ef}Bi/R^2 + (bU)^2/a_{ef}]z^2/a_{ef}} - 1]. \qquad (18)$$

Let us consider the special case in which heat transfer from the system by the gas is negligibly small and heat losses occur only through the external cylindrical surface. Equation (18) becomes simpler:

$$\varphi_{Bi} = \frac{\tau_m}{\tau_m^*} =$$
$$= \frac{1}{2Bi}\left(\frac{R}{z}\right)^2\left[\sqrt{1 + 4Bi\left(\frac{z}{R}\right)^2} - 1\right], \qquad (19)$$

which shows that the correction φ_{Bi} is a function of the two groups z/r and Bi. With increasing values of Bi and z/R the magnitude of the correction becomes increasingly different from 1 (Fig. 1). It should be noted that because of the relatively high thermal diffusivity of the fluidized bed, Bi is usually small (Bi < 1), even in the case of a large apparatus. It is therefore interesting to assess the conditions when the loss of heat from the bed to the external cylindrical surface can be neglected. Usually, the accuracy of the determination of the thermal diffusivity does not exceed 5%; it is therefore permissible to take φ_{Bi} not less than 0.95. With this condition Eq. (19) gives the inequality

$$\left(\frac{z}{R}\right)^2 Bi \leq 0.14. \qquad (20)$$

In Fig. 1 this condition is represented by the shaded part of the graph, which shows the permissible range of variation of z/R and Bi when in processing the experimental results use is made of the simpler relation (17).

Let us now turn to the other limiting case: the column is well insulated and the condition (20) is satisfied, but heat is carried away by the gas. In this case

$$\varphi_U = \tau_m/\tau_m^* = 2(a_{ef}/bUz)^2[\sqrt{1 + (bUz/a_{ef})^2} - 1]. \qquad (21)$$

The magnitude of the correction varies from 1 to 0 decreasing with increasing bUz/d_{ef}, as may be clearly seen from Fig. 2 where (21) is represented graphically.

If the correction φ_U differs little from unity ($1 > \varphi_U > 0.95$) then, obviously, the effect of heat loss via the fluid flow may be neglected and the following inequality applies:

$$bUz/a_{ef} \leq 0.47. \qquad (22)$$

If this condition is satisfied the well-known relation (17) can be used. The error thereby introduced is less than 5% and remains well within the required range of test accuracy. The region where the condition (22) is satisfied is indicated in Fig. 2 by shading.

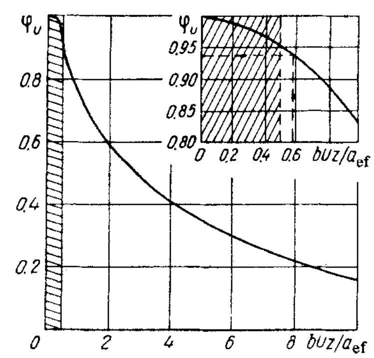

Fig. 2. Dependence of the time needed to reach the temperature maximum φ_U on the group bUz/d_{ef}.

We now illustrate the above considerations by an example. It is required to measure the effective thermal diffusivity of a fluidized bed along the vertical in a 1-m diameter cylindrical apparatus in which the height of the bed is z = 1.5 m. We use the modified method of instantaneous heat sources [1]. According to the literature data $a_{ef} = 20$ cm²/sec, U = = 1.5 m/sec, α = 20 W/m²·°C. The volume heat capacity of the bed and gas are 10^6 and 500 J/m³ · °C, respectively. It is easy to see that these conditions fail to satisfy Eq. (22):

$$bUz/a_{ef} = 500 \cdot 1.5 \cdot 1.5/10^6 \cdot 20 \cdot 10^{-4} > 0.47,$$

while condition (20) is satisfied:

$$\mathrm{Bi}\left(\frac{z}{R}\right)^2 = \frac{\alpha\cdot 2R}{a_{ef}c_{pa}\gamma_{pa}(1-\varepsilon)}\left(\frac{z}{R}\right)^2 = \frac{20\cdot 1}{20\cdot 10^{-4}\cdot 10^{6}}\left(\frac{1.5}{0.5}\right)^2 < 0.14.$$

For this reason in measuring a_{ef} relation (17) can be used with a correction which takes into account the heat carried away from the bed by the fluidizing gas obtained from Fig. 2 or relation (21).

NOTATION

a_{ef}—effective thermal diffusivity of fluidized bed; b—ratio of volume heat capacities of bed and gas; R—radius of apparatus; c, c_{pa}—specific heats of gas and particles; U—gas velocity; z, r—coordinates; t, Θ—temperatures; τ—time; τ_m—time needed to reach the temperature maximum; α—coefficient of heat transfer from cylindrical surface to surrounding air; φ_{Bi}, φ_U—corrections to τ_m; γ, γ_{pa}—densities of gas and material; ε—voidage.

REFERENCES

1. V. A. Borodulya and A. I. Tamarin, IFZh, no. 11, 1962.
2. A. V. Luikov, Theory of Heat Conduction [in Russian], Moscow, 1952.
3. D. D. Ivanenko and A. A. Sokolov, Classical Field Theory [in Russian], Moscow-Leningrad, 1949.
4. I. G. Aramovich and V. I. Levin, Equations of Mathematical Physics [in Russian], Izd. Nauka, 1964.

10 April 1965

Institute of Heat and Mass Transfer
AS BSSR, Minsk